

Overview

The slope-area technique provides an estimate of the discharge in a stream, and is used when other more accurate methods such as current meters or floats are not possible. Its chief advantage is for flood gauging, where an estimate of the peak flow can be made without the necessity to be on site at the time or to put equipment and personnel at risk in extreme conditions. With this technique cross-sectional area, hydraulic radius and water-levels are measured at several sections along a reach. The roughness of the channel is estimated, and the discharge determined by using the Manning open channel flow equation.

There are two other methods of indirect flow determination that may be useful. Refer to the Training Notes on the Culvert Method and the Contract Opening Method for more information.

Selection of Reach

The reach should:

- Be straight.
- Contain the flow without overflow at the stages measured. Have as clear a waterway as possible.
- Have relatively uniform cross-sections and preferably be converging; the increasing velocity through the reach minimises deposition of bed load and ensures a more stable bed profile.
- Be sufficiently long so that uncertainties in slope measurements will not be significant. The fall in water surface elevation down the reach shall be at least ten times the expected error in defining water-levels and levelling them. This is usually a minimum of 5 flood widths
- Have flood marks of good quality and quantity, in the case of measurements made following an event.
- Be sufficiently far away from sharp bends, either above or below, so that water-levels on each bank will not differ greatly in height.
- Be where significant backwaters or tidal effects do not occur.

Measurement Procedure

- Select at least two and preferably three or more cross-sections on the reach, and mark them with stakes or the like for future reference.
- Peg water-levels during or as soon as possible after the flood, as flood marks will be clearer within a few days of the event. Define an actual or estimated water level using a 50 x 50 mm survey peg driven in obliquely so that the uppermost corner defines the level. "Flag" this peg with a 25 x 25 mm survey stake coloured with spray paint.



- Accurate measurement of water-levels is most important as the slope estimate is sensitive to small errors in water-level measurements. Therefore, consistency is more important than absolute accuracy, and generally similar locations should be sought and similar types of flood marks used.
- Locate as many high water marks down the reach as possible, in order to aid in the interpretation of the profiles.
- Defining water levels from flood marks is difficult. Surge and wave action tend to deposit flood marks higher than the mean maximum level, and backwaters or sheltered areas should be sought to minimise these effects. Be wary of where irregularities in the bank profile (humps, hollows, large rocks and logs) may produce inconsistent water levels.
- Deposits of silt and sand tend to be better marks than debris, and flood marks on the ground are generally more reliable than those in trees or bushes. High water velocities may have caused the vegetation to bend over during the flood and then straighten up as the flood levels receded. As much as possible, be consistent in the types of flood mark and their location, remembering that consistency is more important than absolute accuracy for the slope measurement.
- A flood level derived from an area of ponded water within vegetation will have been little affected by surge and should give an accurate level. However, if at another section the flood level is derived from flood marks where the surge would have elevated them, the measured fall will be in error by the amount of the surge. On many reaches the fall will not be large relative to the surge, and a large uncertainty can arise through this.
- On a reach intended for a number of slope-area gauging's, crest-stage indicators will be worthwhile. These will give conveniently read and relatively accurate water levels, and thus more accurate slope measurements.
- With rapidly changing stage (the majority of our rivers) it is necessary to peg the reach simultaneously if possible.
- Measure the areas of the cross-sections, using levelling techniques and gauging equipment for the waterway. Measure a sufficient number of points (at least 20) up to and well beyond the maximum waterway.

Measure the distances between the cross-sections with either survey techniques such as stadia, or a tape or tagline if possible. As the cross-sections may not be completely parallel, either measure along both banks and take the mean, or measure along the centre-line

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Calculation

Calculations shall be carried out as set out on the slope-area gauging card, WS Form 4E. The card shall be completed in all details and numbered and filed as for other types of gauging's.

Determination of Uncertainty

Like other methods of gauging, it is not possible to predict the errors exactly, but the individual uncertainties can be estimated and combined to provide a useful measure of the confidence with which the result can be used. For the general method, refer to the Training Note Determination of Uncertainty.

Uncertainties of the various components shall be estimated or, where applicable, the standard deviations shall be calculated and all values combined to give a total uncertainty that can be filed.

The uncertainties of the components may be calculated by:

- Uncertainties in length of reach; if this can be measured independently 3 or more different times, calculate the standard deviation σ of these measurements and double it. Otherwise the uncertainty can be taken as one-half of the interval within which the error is estimated to lie (ISO 5168). (This interval is termed the tolerance.)
- Uncertainties in width; estimate the possible uncertainty in measuring this parameter.
- Uncertainties in depth; estimate the uncertainty in depth according to the method used, and with due regard to variations in water-level during the measurement.

Uncertainties in the roughness coefficient; estimate the likely uncertainty in the Manning nCombine the uncertainties and record



PART IV - FURTHER CALCULATIONS

$$K = \frac{\overline{A} \times \overline{R}}{n}^{\frac{2}{3}}$$

Converging

Q

-	(2.gl) ^{1/2}	× к ×	s ^{1/2}
	((K)2	(K)2)1/2
	(2.gl +	(A2) -	(\overline{A}_1))
	()

g = 9.81I = Length of Reach s = Slope

Diverging

Q =	(4.gl) ^½ × I	$< \times s^{\frac{1}{2}}$
(K)2)½
(4.gl	$+$ (\overline{A}_{2}) $-$ (A_{2})	Ā1))
()

				at	
River No	******		•	ference:	
MEASUREME	NT DATA.		Date:		
Water-levels					
				at (time)	
				on	
				on	
				on	
Details of any	v other discharge	meas. made	:		
				COMPUTED DATA:	
	STAGE REA	ADINGS		Discharge:	l/s.
Time	Chart	Well	River	Stage Ht. change nil/	m
				Rate of rise/fall	mm/
	Meas. began			Area	m²
				Width	m
				Max. Depth	m
				Max. Surf. Vel.	m/s
				Mean Vel.	m/s
				Wet. Perim.	m
			1	Hvd. Bad.	m
				Hyd. Rad	m
					m
	Meas. ended			Slope	m
	Meas. ended			Slope	m

NATIONAL WATER AND SOIL CONSERVATION AUTHORITY

WS Form 4E

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Site No.



PART I FIELD DATA

X-Sect No. and Location	Water Surface R.L.			Distances Between	
	Left Bank	Right Bank	Mean or Preferred	X-Sects (on centreline)	Remarks
				_	

PART II - LOCATION DIAGRAM

(Show X-Sects, BM'S, etc. and all measurements.)

PART III - CALCULATIONS

(Use preferred combinations of two sections, but if suitable include data from a middle section for calculations of \overline{A} and $\overline{R}.)$

A ₁	$\frac{A_1}{WP_1} = \dots$
A ₂ =	$\frac{A_2}{WP_2} = \dots$
A ₃ =	$\frac{A_3}{WP_3} = \dots$
Ā =	R =
	\overline{R} $\frac{2}{3}$ =
Mean R.L. Top Sect = Mean R.L. Bottom Sect = Fall =	
Slope = Fall =	$\therefore s^{\frac{1}{2}} =$
Use n =	REMARKS
$Q = \frac{\overline{A} \times \overline{R}^{\frac{2}{3}} \times \overline{S}^{\frac{1}{2}}}{n}$	
=	
	Calculated by:
57397J—2,000/7/86MK	Checked by:



SLOPE-AREA MEASUREMENT

• Slope-area method: The slope-area method is used where there have not been measurements of discharge to provide a rating curve. It uses surveyed cross-sectional information and the Manning and Chézy friction laws, together with slope, to calculate the discharge *approximately*. This is used to calculate Q_{Max} after the pegging of a water level during an inspection throughout a flood. From this information the slope, cross-sectional area and wetted perimeter can be calculated, and Q_{Max} computed with either the Manning formula or Chézy formula (more often the Manning formula). To do this however, roughness coefficients *must be known*, such as Manning's n

$$Q = \frac{1}{n} \frac{A^{5/3}}{R^{2/3}} \sqrt{S}$$

in the formula where A is the area, R the hydraulic Radius, and S the slope. It is ballpark at best. In most cases, the value of n is unknown (it's not directly measureable). To find values of n, simply rearrange the equation:

$$n = \frac{\sqrt{S}A^{\frac{5}{3}}}{R^{\frac{2}{3}}Q}, S \neq 0, Q \neq 0, R \neq 0, and \frac{\sqrt{S}A^{\frac{5}{3}}}{QR^{\frac{2}{3}}} \neq 0$$

Often slopes are ignored during calculations, and an assumed value of n is made from a reference table. If all values are known, except S, the Manning's formula can be rearranged for S:

$$S = \frac{R^{\frac{4}{3}}n^2Q^2}{A^{\frac{10}{3}}}, n \neq 0 \text{ and } R \neq 0 \text{ and } -\frac{A^{\frac{5}{3}}}{nR^{\frac{2}{3}}} \neq 0$$

• Q = AV: Where a slope area calculation is absent, the second port of call is a simple Q=AV calculation. This can be achieved in any number of ways, but is often purely extracted from the gauging information and assumptions are made as to where both A and V lie in respect to η_{max}

Principle of the Method

It will be evident to the field hydrologist that the velocity in a reach will be related to the slope of the channel and to the bed friction, which is due to the roughness of the bed material. The Manning equation below gives a relationship for these, together with a factor for the depth of flow. These values are calculated for the reach, rather than a single cross-section, with the values thus being averaged between the two or more cross-sections.

The version of the Manning equation employed (see note below) to compute the velocity is:

$$V = \frac{R^{2/3} S_f^{1/2}}{A R^{2/3} S_f^{1/2}}$$
$$Q = \frac{R^{2/3} S_f^{1/2}}{A R^{2/3} S_f^{1/2}}$$

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and as discharge = velocity x area; where: V is mean velocity (m/s) Q is discharge (m³/s) R is hydraulic radius = A/P (m) A is mean cross-section area (m²) P is wetted perimeter (m) S_f is the hydraulic gradient (m/m); for definition see Figure 1. *n* is the Manning roughness coefficient.

- a. The hydraulic radius, R, is calculated from gauging or cross-section data by RICODA. As noted above, it is obtained by dividing the area of the cross-section by the wetted perimeter. The latter is the length of the boundary between the water and the bed and banks. For each partial section of the cross-section, it is calculated from the width and the difference in depth of the adjoining verticals.
- b. A number of methods are employed in the literature to describe the slope. The energy slope is sometimes taken to be equivalent to the friction slope, but this does not allow for energy losses due to expansion of the reach. S_f is used here in accordance with the methods described in Dalrymple and Benson (1966).

The Friction Slope S_f



Where the channel neither converges nor diverges the hydraulic gradient (S_f) can otherwise be termed the slope of the energy gradient (S_e) , and is identical to the water surface slope. In other cases though, the water surface is affected by changes in velocity head and by other losses, and S_f must be calculated as outlined below.







Converging Reaches

In a converging reach the cross-sectional area of the downstream section will be smaller than the upstream one. S_f should be computed according to the formula

$$S_f = \frac{S + (V_1^2 - V_2^2)}{2gL}$$

where

S is the water surface slope (m/m) V_1 is the mean velocity at the upper section (m/s) V_2 is the mean velocity at the lower section (m/s) **g** is the gravitational constant, 9.81 (m/s²) **L** is the length of the reach (m).

Because the velocities at the upper and lower sections are not known, the method of successive approximations must be used. This entails assuming a value of Q, dividing by the respective cross-sectional areas to get V_1 and V_2 , computing and applying it to the Manning formula to derive Q. The process is repeated using successive values of Q until the assumed and computed values are the same.

Assuming a high and then a low value for Q, then graphing assumed Q against calculated Q will allow a more precise estimate to be made for the third calculation.

Diverging Reaches

The process is similar to that for converging reaches. The formula is

$$S_f = \frac{S + k(V_1^2 - V_2^2)}{2gL}$$

where k is a correction factor for the expansion losses. A value of 0.5 is sometimes used for k, but this is questionable. Consequently, and because diverging reaches tend to be unstable, they are best avoided.

Estimation of Manning's n

Even when the slope and the channel geometry can be measured accurately, there will normally still be substantial uncertainty with the estimation of the roughness coefficient, *n*. The higher the roughness, the higher the *n*. In gravel bed rivers it is typically in the range 0.020 to 0.050, and tends to vary with discharge.

The best way to define an appropriate n is to carry out a number of current meter gauging's at the site at a range of flows and calculate the n from the transposed Manning equation:



$$n = \frac{AR^{2/3}S_f^{1/2}}{Q}$$

Normally, as Q will be the unknown value of interest, it will not be possible to do this, particularly for high water-levels. However, other, usually lower-stage, gauging's at that site may be able to be used to calculate n values. As it is most likely that n will vary with stage, a stage n curve should be compiled and extended by eye in order to estimate n for higher flows.

The above method is recommended, as it is usually the only way of deriving n with any degree of confidence. In many cases, however, there will be insufficient data to extend the stage-n curve with confidence, and more approximate methods will need to be employed. These methods consist of:

- Reference to publications that provide examples of reaches with measured *n* values along with slope and bed material data and colour photographs of the reaches. Hicks and Mason (1991) provide New Zealand examples and Bames (1967) gives U.S. ones. Arcement and Schneider (1989) give examples for vegetated flood plains.
- Where the slope exceeds 0.02, the use of an equation from Jarrett (1984) which estimates *n* using slope and depth of flow parameters. The metric version of his equation is:

$$n = 0.32 \text{ S}_{\text{f}}^{0.38} \text{ R}^{-0.16}$$

- a. This equation assumes that the size of bed material is related to slope, and relies on the latter parameter to take account of the former. This will hold for alluvial channels, but probably not, for instance, to bedrock ones.
- b. The equation has been checked against data for some New Zealand rivers (Hydrology Centre, 1988) and found to be adequate.
- Estimation by eye and personal experience. This will be inadequate. Even very experienced USGS personnel (Hicks, Jarrett pers. comm., in J.K. Fenwick, 1991) have found this to be so.

When using any of the above techniques, the relevant publications should be obtained and studied.



Calculation

Calculations shall be carried out as set out on the slope-area gauging card, WS Form 4E. The card shall be completed in all details and numbered and filed as for other types of gauging's.

The example given in Figure 2 illustrates the use of this form. The difference in the cross-section area between sections 1 and 2 indicates a diverging reach. In this example discharge is calculated using both the standard formula for straight reaches (part III on the gauging card) and the modified formula for diverging reaches (part IV). The diverging reach formula gives a discharge about 8% higher than that derived by the standard formula.

6.3.1 A C Hopkins' Method

The following examples illustrate slope-area discharge calculations by both the 'direct' method employed on WS Form 4E and the 'trial and error' method.

These calculations are a somewhat simplified version of the appropriate hydraulic formulae in that both the upstream and downstream cross-sections are used in their entirety, without any subdivision, or the consequent derivation of multiple adjustment coefficients for each. A single value of Manning's n is selected as applicable to the reach as a whole and the Coriolis coefficient α , used in the calculation of velocity head, is assumed to be 1.

Physical data for a <u>converging</u> reach:

Upstream cross-section area	$A_1 = 1000 \text{ m}^2$
Downstream cross-section area	$A_2 = 800 \text{ m}^2$
Mean cross-section area	$A = 900 \text{ m}^2$
Upstream hydraulic radius	$R_1 = 5.5 m$
Downstream hydraulic radius	$R_2 = 4.5 m$
Mean $R = 5.0$	R - = 2.924
Length of Reach	L = 400 m
Slope of Water Surface	$S_s = 0.0004$
Assessed Manning's n	n = 0.028
Gravity Constant	$g = 9.8 \text{ m/s}^2$
Conveyance of Reach $k = (A \times R^{2/3})/n$	k = 93986

Trial and Error Method

As the reach is converging by 200 m² (i.e. degree of convergence is 20%) it is necessary to adjust the slope, S_s , to obtain the slope of the energy gradient,

 $S_e = S_s \text{ - } \Delta S$

1. First approximation of Q, using the slope of water surface S_s thus

$$\mathbf{Q} = \mathbf{k} \, \mathbf{S}_{\mathbf{s}^{1/2}}$$

 $= 93986 \times 0.02$



 $= 1880 \text{ m}^{3/\text{s}}$

Because of the degree of convergence this discharge will be much too high because the slope S_e is much 'flatter' than the slope S_s .

2. So, assume $Q = 1500 \text{ m}^3/\text{s}$, and derive S_e .

$$\Delta S = \frac{V_2^2 - V_1^2}{2gL}$$

$$= \frac{(1500 / 800)^2 - (1500 / 1000)^2}{2 \times 9.8 \times 400}$$

$$= 0.000161$$

$$S_e = S_s - \Delta S$$

$$= 0.0004 - 0.000161$$

$$= 0.000239$$
Therefore: $S_e^{V_2} = 0.0155$

 $Q = k S_e^{\frac{1}{2}}$ = 93986 × 0.0155 = 1453 m³/s

Therefore the assumed $Q = 1500 \text{ m}^3/\text{s}$ is too high

3. So, assume $Q = 1400 \text{ m}^3/\text{s}$, and derive S_e .

$$\Delta S = \frac{V_2^2 - V_1^2}{2gL}$$

$$=\frac{(1400 / 800)^2 - (1400 / 1000)^2}{2 \times 9.8 \times 400}$$

$$= 0.000141$$
$$S_e = S_s - \Delta S$$



= 0.0004 - 0.000141

= 0.000259

Therefore: $S_e^{\frac{1}{2}} = 0.0161$

 $Q = k S_e^{\frac{1}{2}}$ = 93986 × 0.0161 = 1513 m³/s

Therefore the assumed $Q = 1400 \text{ m}^3/\text{s}$ is too low.

4. Draw a graph of values of assumed Q versus computed Q and draw in the 'equal value line', then plot in the two points we have of the assumed values and their corresponding computed values and join these with a straight line. The intersection of this with the 'equal value line' will indicate the correct value, $Q = 1470 \text{ m}^3/\text{s}$.

Note that had the reach been diverging the calculation would have been modified as follows:

$$S_e = S_s + \frac{1}{2}\Delta S, \text{ and } \Delta S = \frac{V_1^2 - V_2^2}{2gL}$$

Because the slope S_e is much steeper than the slope S_s . The remaining steps of the calculation follow the same form.

Direct Method

The formula is:

$$Q = \frac{(2gL)^{1/2} \times k \times S_s^{1/2}}{\left[(2gL) + (k/A_2)^2 - (k/A_1)^2\right]^{1/2}}$$

Therefore:

$$Q = \frac{\sqrt{2 \times 9.8 \times 400} \times 93986 \times \sqrt{0.0004}}{\sqrt{(2 \times 9.8 \times 400) + (93986 / 800)^2 - (93986 / 1000)^2}}$$
88.5438 \times 93986 \times 0.02

$$=\frac{1}{\sqrt{7840+13802-8833}}$$

$$=\frac{166438}{\sqrt{12809}}$$



 $= 1471 \text{ m}^{3/s}$

Check by the Trial and Error method.

Assume $Q = 1471 \text{ m}^3/\text{s}$, and derive S_e .

$$\Delta S = \frac{V_2^2 - V_1^2}{2gL}$$

$$= \frac{(1471/800)^2 - (1471/1000)^2}{2 \times 9.8 \times 400}$$

$$= \frac{1.217}{7840}$$

$$= 0.000155$$

$$S_e = S_s - \Delta S$$

$$= 0.0004 - 0.000155$$

$$= 0.000245$$
Therefore: $S_e^{\frac{1}{2}} = 0.01565$

$$Q = k S_e^{\frac{1}{2}}$$

$$= 93986 \times 0.0161$$

 $= 1470.88 \text{ m}^{3}/\text{s}$

Note that had the reach been diverging the only change required in the formula for direct calculation is the substitution of the factor 4gL for the factor 2gL.

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hori	zons	Hydrolo	gy Operatio	ns	(M
regional			Manual		hoi	
Figure of a area using card	SLOPE-AREA DISCHARGE TIR.IT.C.A. River No. MEASUREMENT DATA: Water-levels pegged: directly, by M.M.K.K.M.A. J. M. Star: Levels, Reac. No. 324 Cross-sections surveyed by: C.C.M.C.J. A.M. Slope data obtained by: C.2.R.C. D.C.K.H. method: Details of any other discharge meas. made: STAGE READINGS	Market On 11: 9:88 Market On 11: 6: 5 5 Market COMPUTED DATA: Discharge: 80600 (/s. Stage Ht, change nil/ — m Rate of rise/fail — mm/h	PART I FIELD DATA X-Sect No. Water Surface R.L. and Left Right Mean or Location Bank Bank Preferred 1 100-000	Distances Betwee X-Sects (on centreli 38 - 9 24 - 3 Foral Ditance: 6 asurements.)		2 - An example slope- gauging the gauging Form 4E.
	AT THE SOUTH BOUND	Width m Max. Depth m Max. Surf. Vel. m/s Mean Vel. .136.1 Max. .023.6 Stope .0.02.4 n .0.02.24 n .0.02.24 n .0.02.24 n .0.02.24 n .0.02.25 Water Level R.L.	preve uncorrection of the second of the seco	2400		
	PART III — CALCULATIONS (Use preferred combinations of two secti section for calculations of A and R.) A ₁ — <u>%6:66.0</u>	ons, but if suitable include data from a middle	PART IV – FURTHER CALCULATIONS $K = \frac{\overline{A} \times \overline{R}}{0}^{\frac{3}{2}} + \frac{60 \cdot 29}{0 \cdot 25} \times \frac{911}{0}$	g = 9.81 I = Length of	Reach	
	A ₂ =	$\frac{A_1}{WP_1} = \frac{0.737}{0.980}$ $\frac{A_2}{WP_2} = \frac{0.980}{0.980}$	n 169.3 Converging $Q = (2.gl)^{\gamma_2} \times K \times s^{\gamma_2}$	s = Slope $A_1 = 46.6\circ$ $A_2 < 73.98$ Diverging $Q = (4.gl)^{\frac{1}{2}}$		
	A ₃ = <u>73.990</u> Ā = 59.26	$\frac{A_3}{WP_3} = \frac{1.001}{\overline{R}}$ $\overline{R} = \frac{0.906}{0.936}$	$\frac{(K_{2})}{(2.gl + (A_{2}) - (A_{1}))}$ (2.gl + (A_{2}) - (A_{1})) ($\frac{((K)2}{(4.g) + (A_2)} - ($	(K)2)½ (A,))) 569-3 < 0 0509	
	Mean R.L. Top Sect = $/00.00$ Mean R.L. Bottom Sect = 99.82 Fall = $0./64$	6		- <u>3972</u> - 42:38		
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